Infrastructure Restoration - A Scheduling Game with Precedence Constraints. Benchmark Instances Generation and Evaluation

Visit Conclusion Report for the Mobilita 200 Grant Programme
Date of Report: 12.2.2014

The details of the visit:
- Student: Petr Kalina, Intelligent Systems Group, Dept. of Cybernetics
- Venue: Optimization Research Group, National ICT Institute of Australia, Anzac Parade 385, Kensington, Sydney
- Host: Prof. Toby Walsh
- Dates: 20.12.2013 - 2.2.214 (actually the stay was extended to 14.2. by NICTA and the participant)

Programme:
- Cooperation within the Computational Disaster Management Project - Infrastructure Restoration - A Scheduling Game with Precedence Constraints. Benchmark Instances Generation and Evaluation
- Seminars and networking activities
- Participation on the Summer School of Optimization

Brief Outline of The Visit:
NICTA is one of the leading Australian research organizations with a very good worldwide renown. Specifically the Optimization Group is a world-class research group with people like Pascal van Hentenryck or Toby Walsh being recognized authorities in their fields.

I was able to familiarise myself with the projects, operations and people within the group, participating on numerous seminars, networking and social events. In particular, the Summer School of Optimization was very interesting for me, being a intensive course on interesting topics in optimization. The detailed programme is available [here](#) and covered topics e.g. Constraint Programming and Modelling, Integer and Mixed Integer Programming, Uncertainty in Optimization, Column Generation, ANswer set programming and many more. The lecturers - Pascal Van Hentenryck, Martin Savelsbergh, Toby Walsh, Guido Tack are top tier scientists in their field.

Apart from the networking and study potential, the rest of the time was dedicated to collaboration on the above mentioned topic. Details follow.
Scope of the Research:
We consider a specific class of scheduling problems inherent to situations where multiple parties contribute to restoration of a common infrastructure e.g. electricity or a gas network. A particular part of the infrastructure is considered to generate profit for its owner that is proportional to the time for which the service within that part of the infrastructure is restored. The time of restoration corresponds to the actual repair of the part of the infrastructure. However, the service also cannot be restored until some other parts of the infrastructure - potentially supplying necessary assets like electricity or gas for the considered part - are repaired as well. These may be potentially owned by other parties with selfish interests, providing for a competitive settings within the overall restoration problem.

We present a model of a scheduling game with precedence constraints abstracting the features of the mentioned scenario and analyze its inherent features. The overall aim of this work is to analyze the theoretical boundaries for the possible losses in the inherent real world situations and is a part of a wide range of activities performed at NICTA concerning the computational disaster management.

The Interdependent Scheduling Game is defined as follows:
- N players independently schedule their sets of tasks (corresponding to repairs of individual parts of the infrastructure owned by the particular player)
- for each task a profit-per-time-unit (denoted as reward) and a duration is defined
- for each tasks a set of prerequisites is defined (corresponding to the topology of underlying infrastructure)
- a task is considered active (corresponding to the operation being restored within the corresponding part of the infrastructure) at the maximum of the following two times: (i) the time when it is scheduled and (ii) the time when all of its prerequisites are scheduled (e.g. the time last of these is completed)
- profit extracted from an individual task is then proportional to the reward of the task and the interval between the task becoming active and the end of schedule (which is considered common for all players)
- for simplicity we consider a model with each task having a duration of 1 and each player scheduling the same number of tasks. we show elsewhere that this simplification is equivalent to settings with tasks with general integer durations.
- the notations and exact formalization of the model is available elsewhere and is currently being worked on

We consider the game theoretic aspects of the problem. As outlined above, there is a disparity between the overall interest of maximizing the overall revenue from the whole infrastructure and the selfish interests of the individual players. In multi-agent literature, the earlier is typically denoted as maximizing the social welfare. The latter on the other hand is usually captured by the notion of a game equilibrium e.g. nash equilibrium - corresponding to the situation when the game is stable in some sense with none of the players having the incentive to unilaterally change its strategy. The nash equilibrium in particular corresponds to a situation, when, given a
particular game profile (the set of schedules of particular players), each schedule corresponds
to a best response of the player to the schedules of the other players. This means that neither of
the players can increase his profit without simultaneously coordinating with some other players.

Given a game thus the relation between the equilibria and the social welfare is one of the
important markers characterizing the impact of selfish behavior of the players to the social
welfare. Two commonly used characteristics with this respect are the Price of Anarchy (PoA)
and Price of Stability (PoS) [ref] corresponding to the ratio between the utility extracted from the
social welfare maximizing (globally optimal) solution and the worst and best nash equilibria
respectively. Thus they correspond to the worst and best outcomes resulting from players'
selfish choices of strategies.

The contribution present within this work therefore corresponds to the experimental evaluation
the of Price of Anarchy and the Price of Stability (ref: E. Koutsoupias, C. H. Papadimitriou -
Worst­case equilibria) for the infrastructure restoration scheduling game, providing pointers
towards the theoretical analysis of the problem.

Benchmark Instances Generation and Evaluation
We present two contributions. The first contribution consists of developing an infrastructure for
(i) solving the social welfare optimization (SWO) problem and (ii) the identification of the extreme
pure nash equilibria (PNE) for a particular problem instance. Secondly we present several sets
of randomly generated problem instances with different attributes and analyze the SWO and
ENE in order to assess the game theoretic properties of the problem.

The SWO problem is NP-hard in general. We have proven that even the 2 player problem with
general acyclic dependencies and uniform rewards is NP-hard. For specific problem variants
e.g. 2 player problem with uniform rewards and with chain dependencies or bipartite
dependencies (only dependencies between tasks owned by different players) being considered a
complete polynomial algorithm exists.

SWO problem
The solution to the SWO problem can be provided in a straightforward manner by expressing the
problem as constraints optimization programming (COP) problem and solving it using one of the
available COP solvers. For the purposes of the experimental assessment of the theoretical
properties of the problem such an approach is satisfactory. The solver for the SWO subproblem
was developed using the MiniZinc [www.minizinc.org] modeling language and the available
underlying COP solvers.

Extreme PNE Problem
The problem of existence of pure nash equilibrium (PNE) is an open question for the considered
problem. In potential (ordinal) games the existence of a PNE is certain and it can be identified
using best-response dynamics. However, the game is a potential game if and only if the best
responses cannot cycle (http://www.cer.ethz.ch/research/research_seminar/ordinalgames.pdf), which is unfortunately not the case for the considered problem - to the contrary we have proven that best responses can cycle even in relatively simple cases. However co conjecture holds that given some limitations to player choices (e.g. exploring all best response paths) the dynamics shall yield PNE in general case.

For specific classes of non-potential games, efficient algorithms for identification of PNE exist - namely for games allowing some compact representations e.g. constraint games, graphical games or action graph games [Daskalakis - Computing Pure Nash Equilibria in Graphical Games via Markov Random Fields, Thi-Van-Anh Nguyen - Constraint Games: Framework and Local Search Solver, Bhat, Leyton-Brown - Computing Nash Equilibria of Action-Graph Games].

The identification of the extreme pure nash equilibria (PNE) is an even more complex problem as even using best response dynamics or local search doesn’t guarantee identifying a global optima. To our knowledge, an efficient algorithm for identifying the extreme PNE fitting for the considered scheduling game is not known.

Considering a game in normal form, the general algorithm consists of traversing all possible strategy profiles and validating whether for each individual player his strategy within the examined profile corresponds to the best response to the strategies of all the remaining players.

Given an efficient optimization algorithm is available for the SWO, this can be easily verified by applying this algorithm to a constrained subproblem with the additional constraints corresponding to fixing the particular strategies of the other players within the examined profile. Moreover, given that multiple equilibria a exist for a given problem, the search tree can be efficiently pruned by omitting solutions with social welfare not in the feasible ranges with respect to the already identified equilibria.

Similar approach is used in the presented solver. The solver is based on embedding the above mentioned constraint optimization based SWO solver within a Java framework. The used algorithm consist of generating all possible combinations of schedules for the N-1 first players, then computing the set of all possible best responses for the Nth player using a modification of the SWO algorithm. Then, for a particular best response of player N, a set of subproblems is solved consisting of computing the utilities of the unilateral best responses for each of the remaining N-1 players. The profile corresponds to a PNE if the utility of the best responses matches the utility of the examined solution.

Benchmark Instances Generation and Key Attributes
We considered games with 2 players and schedules of length 4 with acyclic dependency graphs. We considered the two key characteristics of the problem - the topology of the underlying dependency graph and the distribution of rewards for the individual tasks. Following notation is used in descriptions of the various graph topologies:
• inter-dependency corresponds to a dependency (an edge in the dependencies graph) between two tasks each owned by a different player
• intra-dependency corresponds to a dependency between two tasks owned by the same player
• indegree (outdegree) of a task (vertex within the dependency graph) corresponds to a number of tasks the task depends on (that depend on the task)

We considered the following general settings with respect to the topology of the underlying dependency graph:
• unitary dependencies (or chain dependencies): outdegree(t) <= 1 and indegree(t) <= 1 for each task t
• bipartite dependencies: no intra-dependencies are present
• one-way dependencies: there exists a numbering of the players such, that for each interdependency the number of player owning the source vertex is lower than the number of the player owning the target vertex
• general dependencies - no restrictions on the dependency graph are given (apart from being acyclic, which is a natural requirement given the nature of the modelled real-world problem)

We considered the following general settings with respect to the distribution of the rewards:
• uniform rewards
• general rewards (a randomly generated rewards in the range of 1..10)

A particular configuration is thus denoted by a pair [graph configuration, rewards configuration].

We considered the feasible combinations of the above configurations.
For each configuration a set of 20 instances was generated and solved. The results are available within the provided bundle.

Evaluation of Results
The results are summarized below. The aim of the experiments is to assess the theoretical properties of the various problem variants and provide valuable evidence for further theoretical analysis of the problem.

The values correspond to avg / max values of the PoA, PoS across the problem sets

<table>
<thead>
<tr>
<th>Algorithm Variant</th>
<th>Price of Anarchy</th>
<th>Price of Stability</th>
<th>Conflict free SWO</th>
</tr>
</thead>
<tbody>
<tr>
<td>[bipartite&amp;chain, uniform]</td>
<td>1/1</td>
<td>1/1</td>
<td>yes</td>
</tr>
<tr>
<td>[bipartite, uniform]</td>
<td>1.09/1.33</td>
<td>1.003/1.05</td>
<td>no</td>
</tr>
<tr>
<td>[chain, uniform]</td>
<td>1/1</td>
<td>1/1</td>
<td>yes</td>
</tr>
<tr>
<td>[oneway&amp;chain, uniform]</td>
<td>1.06/1.27</td>
<td>1/1</td>
<td>no</td>
</tr>
</tbody>
</table>
The experimental results (problem instances and the extreme NE and SWO solutions are provided in the corresponding bundle) available at: https://www.dropbox.com/s/mvbalqrcbbmfzcs/solutions.zip. Each of the folders contains a set of text files corresponding to the individual problem instances, a log.txt file containing the summary of the corresponding experiment (including the summary of the relevant results for all the instances) and the mzn folder containing the models for relevant subproblems for the individual problem instances in the MiniZinc modelling language.

Conclusion & Possible Extensions

The experimental results provide pointers towards the theoretical analysis of the problem:

- in all cases we were able to find a pure nash equilibrium
- in all but the simplest variants of the problem even the best nash equilibrium does not result in SW optimal solution. The value of the gap between the two is immaterial as it can be made arbitrarily large by manipulating the rewards of the particular instance

The future experiments may include experimenting with nash dynamics by exploring the paths in the best response graphs and the nash equilibria of the game (Note - the game may be a pseudo-potential game - Schipper - Pseudo potential games - verify that..) and extending the experiments to games with more players.

Another sound topic is the assessment of the key attributes over larger problem instances corresponding somehow to the underlying real-world problem. We reviewed the relevant scheduling literature in search of a suitable benchmark sets. The emergent conclusion is, that while usable benchmarks could be derived from existing problem sets e.g. for precedence graph N-machine scheduling, it’s relevance for the studies real world case is very limited. We argue that relevant instances should rather be generated based on deriving problem from alternative topologies of the underlying network and the distributions of ownership over this network in a
manner that reflects the semantics of the real-world problem. Or by obtaining real world instances from the disaster management cases.