Interdependent Infrastructure Restoration — A Scheduling Games with Precedence Constraints, Thesis Write-up

NICTA Visit Conclusion for Mobilita 200 Programme

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The details of the visit

- **Student:** Petr Kalina, Intelligent Systems Group, Dept. of Cybernetics
- **Venue:** Optimization Research Group (ORG), National ICT Institute of Australia (NICTA), Anzac Parade 218, Kensington, Sydney
- **Host:** Prof. Toby Walsh
- **Dates:** 24.2. 2014 - 22.6.2014

Programme

- Cooperation within the Computational Disaster Management Project of the NICTA ORG[2, 1] on researching infrastructure restoration games — scheduling games with precedence constraints
- Seminars and networking activities
- Work on the thesis write-up titled "Algorithms for the VRPTW based on Agent Negotiation"

1 Brief Outline of the Visit

NICTA is one of the leading Australian research organizations with a very good worldwide renome. Specifically the Optimization Group (ORG) is a world-class research group with people like Pascal van Hentenryck or Toby Walsh being recognized authorities in their fields. The main research areas include general optimization methods e.g. linear, constraint based optimization methods, stochastic and learning approaches to optimization, social choice, game theory and multiagent systems.

I was able to familiarize myself with the ongoing research at NICTA ORG group, meet the people working here as well as the leaders like Haris Aziz, Toby Walsh or Pascal Van Hentenryck, participating on numerous seminars, reading groups, networking and social events. I was able to cooperate with the ORG on a topic related to my thesis in a very interesting way, providing valuable context to my ongoing research. In the meantime, I was progressing with my thesis write-up, receiving valuable feedback from the ORG colleagues.

2 Scope of the Research Collaboration

We consider a specific class of scheduling problems inherent to situations where multiple parties contribute to restoration of a common infrastructure e.g. electricity or a gas network. The model is motivated by large-scale restoration of interdependent infrastructures after significant disruptions. Each particular part of the infrastructure is considered to generate profit for its owner that is proportional to the time for which the service within that part of the infrastructure is restored. However, the service also cannot be restored until some other parts of the infrastructure — potentially supplying necessary assets like electricity or gas for the considered part — are repaired as well. These may be potentially owned by other parties with selfish interests, providing for a competitive settings within the overall restoration problem. We provide a abstract model for the described scenario and a detailed analysis of its fundamental game theoretic and optimization properties including social welfare maximization, computing best responses, Nash dynamics, and the existence of pure Nash equilibria.

2.1 Infrastructure Restoration — a Scheduling Game with Precedence Constraints

We provide a formal model of the game.

Definition 1. *The interdependent infrastructure restoration scheduling game (IRG) for k players and N tasks is defined as a tuple $((T_1, \dots, T_k), G, r)$ where:*

- $T_1 \dots T_k$ *are the sets of tasks corresponding to each of the players with* $|T_i| = N, i = 1 \dots k$. *The tasks correspond to the restoration of a particular infrastructure parts owned by the respective players* $1 \dots k$. *Let* $T = \cup_{i=1}^{k} T_i$. *Also, the duration of all the tasks is considered to be 1.*
- $G = (T, E)$ *is a directed acyclic graph representing the precedence constraints within the problem. Thus* $(t_i, t_j) \in E$, $t_i, t_j \in T$ *corresponds to the task* t_i *being a precedence of the task* t_j *— denoted also as* $(t_i \to t_j)$. *Note that the tasks* t_i *and* t_j *need not to be owned by the same player. A graph corresponding to a transitive precedence relation is considered. We denote* $N_G^-(t) = \{x \in T : (x, t) \in E\}$ *the set of all precedences of the task* t.
- $r : T \to \mathbb{R}_0^+$ *is a reward function assigning a non-negative reward to each of the tasks.*
- *The solution of the game is a set of* k *schedules* $\pi = (\pi_1 \dots \pi_k)$ *— one for each player — corresponding to a particular orderings of the tasks for each player.*
- *Let* $\pi(t)$ *correspond to the position of the task* t *within the corresponding player's schedule. The task* t *is considered active in time* τ *iff* $\tau = max(\pi(t_i)), t_i \in N_G^-(t) \cup t$ *i.e. all of the precedences of* t *and the task* t *itself are scheduled in* τ. *We write* $a(t) = \tau$.
- *The reward for the player* i *within the game is then computed as* $r_i = \sum_{t \in T_i} (N - a(t)) * r(t)$ *eg as being proportional to the (i) time* $(N - a(t))$ *for which the task* t *is active and (ii) the reward* $r(t)$ *associated with* t.
- *The social welfare within the game is then defined as* $W = \sum_{i=1}^{k} r_i$.

2.2 Analysis — a Summary of Contributions

- The above scheduling model was presented capturing scenarios in power restoration after natural disasters.
- *Welfare maximization*: We show that welfare maximization is NP-hard even when the rewards are uniform, $r(t) = 1$ for all $t \in T$, and each player has two tasks. It is also NP-hard when rewards are uniform and there are two players. On the other hand, for one player, welfare maximization can be solved in polynomial time.
- *Computing best responses*: For two players and uniform rewards, a best response can be computed in linear time. Even for general rewards, and only inter-player dependencies, a best response can be computed in polynomial time. On the other hand, finding a *conflict free* (i.e. $\forall t \in T_i : a(t) = \pi(t)$) best response schedule is NP-hard.
- *Best response dynamics and Nash equilibria:* We show that for uniform rewards and two players, best responses can cycle. Despite the possibility of cycles, we show that a pure Nash equilibrium always exists for uniform rewards and two players. On the other hand, if the rewards are not uniform, then a pure Nash equilibrium may not exist.
- *Cooperative game:* Finally, we model IRG as a cooperative game in which the value of a coalition of agents is the maximum welfare they can generate by coordinating with each other. We show that the corresponding cooperative game is not convex.
- *Price of Stability/Anarchy:* The Price of Stability/Anarchy (PoS,PoA) [3] have been studied for alternative variants of the game. In general case the PoA and PoS are unbounded.
- *Mechanism Design:* A simple mechanism based on bargaining has been proposed for 2-player *one-way* precedence games (i.e. $(u \rightarrow v \implies (u \in T_1 \wedge v \in T_2))$ providing for an improved bounds for the PoA and PoS.

2.3 Relevance of the Collaboration with respect to my Thesis

The problem defined and analyzed above is in essence a multi-agent optimization problem. However, so is in fact the multi-agent based approach to solving the VRPTW that forms the main contribution of my thesis. The two presented analyzes are however fundamentally different. In the research being part of our collaboration here at NICTA, we assume a competitive environment and focus mainly on the game-theoretic problem attributes. The research mostly theoretic and is not concerned (at its present status at least) with addressing the real-world applicability of the proposed methods in terms of performance, timeliness of the response in the real-world case, addressing the uncertainty and dynamism of the environment and facilitating an efficient communication and coordination of the agents.

In the VRPTV research being part of my thesis, on the other hand, the focus is exactly the opposite. While the agent formulation of the VRPTW problem results in a social welfare maximization multi-agent optimization problem i.e.

similarly as above, a fully cooperative environment is assumed simplifying the game-theoretic background. On the other hand, for the developed algorithms the focus is put primarily on the efficiency of the algorithms when compared to the existing traditional heuristic and meta-heuristic methods in an effort to assess the real-world applicability of the presented algorithms. Further stressing the point is the analysis of the improvements in terms of execution stability of the multi-agent VRPTW algorithm in a dynamic uncertain environments in case the inherent autonomic features of the multi-agent problem decomposition algorithm are leveraged.

By being able to experience hands-on these two fundamentally different ways of researching what are in essence a multi-agent optimization problems I think my awareness of the positioning of my work within the overall multi-agent optimization field significantly increased. This helped me immensely when formulating the introduction and context for the research underpinning my thesis.

3 Work on the Thesis

Most of my time in NICTA was actually dedicated to writing up my thesis. I was able both to significantly extend the presented work by (i) introducing a novel variant of the presented VRPTW agent-based algorithm and (ii) by significant work being done on the text thesis itself, with over 110 pages being written. The thesis is almost ready for submission. Also, by being able to discuss the thesis material in the context of the ORG group enabled me to gather significant insights from my collaborators arguably adding some new flavor and quality within my thesis.

4 Conclusion

The visit at NICTA spanning to nearly 6 months has been an immensely inspiring period both with respect to my PHD as well as my professional growth as a researcher. By collaborating with a team of top-tier researchers I was able to acquire novel insight in the areas related to my PHD research as well as improve my social, collaboration and language skills. Last, but not least, the opportunity to live and work in Australia and Sydney has been an invaluable life experience. Hereby I therefore also express my extreme gratitude to all the people and organizations making it possible for me to undertake this experience, including the Mobilita 200 programme.

References

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